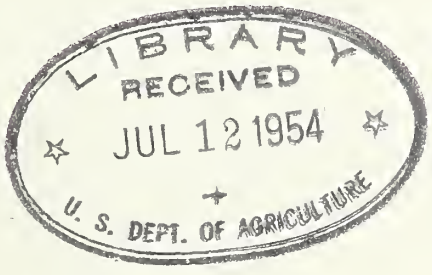
AMORTIZATION OF LOANS

Its Application to Farm Problems

by Ralph R. Botts
AGRICULTURAL ECONOMIST



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Payment of a debt by installments, usually equal in amount, which include both interest on the outstanding balance and a partial payment of principal, is called amortization. Periodic payments are so calculated that by the end of the term, if all payments are made as they come due, the debt will be paid in full. Farmers and lenders are finding that many amortized loans are mutually beneficial. But amortization is sometimes used when it is not the best arrangement. This report explains the principal features of amortization and discusses briefly how amortization can be used to advantage by both borrowers and lenders. Finally, it shows the mechanics of preparing an amortization table.

PLANS OF AMORTIZATION

Standard Plan

Under this plan, the length of the repayment period is decided first; the size of the periodic payment depends upon the length of this period. The periodic payment is used to pay interest and to reduce the principal. This standard plan is most frequently used. Under it, the borrower knows that each payment will be for so many dollars and that at the end of the time, if all payments have been made, the loan will be paid in full. An example of how this plan works is shown in table 1.

Before constructing table 1, a periodic payment was found from table 3 that will exactly amortize the debt in the allotted time (10 years) at the agreed interest rate (5 percent). This payment ($129.50) is the same each year.2/ 3/ Interest at 5 percent is figured on the beginning balance of $1,000 for 1 year. When this interest ($50) is deducted from the periodic payment ($129.50), $79.50 is left for reduction of principal. At the beginning of the second year, the balance is $920.50. At the end of that year, the interest on $920.50 at 5 percent is $46.02, so $83.48 is left for reduction of principal. Therefore, the balance at the beginning of the third year is $837.02. The last payment of $129.50 pays off the debt, with interest for the last period.

1/ Harald C. Larsen contributed some of the ideas used in this report.

2/ Look for 0.12950458 in the 5-percent column of table 3, opposite 10 periodic payments. This figure tells us that payments of 12.95 cents a year for 10 years would pay off a $1 debt, if interest were at 5 percent. At the same rate, therefore, it would take an annual payment of $129.50 to pay off a $1,000 debt in 10 years.

3/ If an annual interest rate applies (such as "5 percent effective"), but payments are to be made more often than once a year, an interest rate equivalent per payment period must be used (from table 6) in constructing the amortization table. See case 2 problems, in appendix.

Table 1.- Standard plan: Amortization of $1,000 loan in 10 years by equal annual installments, with interest at 5 percent 1/

Year	Unpaid principal at beginning of year	Payment at end of year		
		Interest	Principal	Total
	Dollars	Dollars	Dollars	Dollars
1	1,000.00	50.00	79.50	129.50
2	920.50	46.02	83.48	129.50
3	837.02	41.85	87.65	129.50
4	749.37	37.46	92.04	129.50
5	657.33	32.86	96.64	129.50
6	560.69	28.03	101.47	129.50
7	459.22	22.96	106.54	129.50
8	352.68	17.63	111.87	129.50
9	240.81	12.04	117.46	129.50
10	123.35	6.16	123.34	129.50
Total	---	2/ 295.00	2/ 1,000.00	1,295.00

1/ Amount of periodic payment found by multiplying $1,000 by the figure that is opposite 10 periods in the 5-percent column of table 3. This figure is 12.950458 percent, so 1,000 x 0.12950458 = $129.50, the amount of the periodic payment.

2/ Because of rounding, the figures shown in this column do not add to the total shown.

Modified Plan

A plan similar to the standard plan is one in which the periodic payment, rather than the number of installments, is decided upon. The amount of the annual payment is based on some factor, such as ability to pay or mutual agreement, and the length of the repayment period depends on the size of the annual payment. For example, suppose a farmer decides that he can pay $150 a year on a loan. This would cover both interest and reduction of principal. The repayment schedule for a $1,000 loan made on this basis, with interest at 5 percent, is shown in table 2.3/

Interest at 5 percent is calculated on the unpaid balance each year and is deducted from the $150 periodic payment. The rest is used to reduce the principal. Payments on account are made until the loan is paid in full.4/ Any odd amount of principal remaining at the end of the term, together with interest on it for one year, determines the amount of the last payment.

At the end of the first year, the interest on the original amount of $1,000 is $50; thus only $100 of the first payment is left for reduction of principal. The $900 at the beginning of the second year draws $45 interest by the end of the

4/ "Payment(s)" refers to the periodic payment(s), which includes an amount for interest and an amount for reduction of principal.

year; thus the remaining $105 of the second payment is used for reduction of principal. At the beginning of the ninth year, the unpaid balance amounts to $45.09. The interest on this amount for one year, at 5 percent, is $2.25; so only $47.34 is required, as a final payment, to liquidate the debt at the end of the ninth year.

Table 2.- Amortization of $1,000 loan by annual payments of $150, if interest is at 5 percent

Year	Unpaid principal at beginning of year	Payment at end of year		
		Interest	Principal	Total
	Dollars	Dollars	Dollars	Dollars
1	1,000.00	50.00	100.00	150.00
2	900.00	45.00	105.00	150.00
3	795.00	39.75	110.25	150.00
4	684.75	34.24	115.76	150.00
5	568.99	28.45	121.55	150.00
6	447.44	22.37	127.63	150.00
7	319.81	15.99	134.01	150.00
8	185.80	9.29	140.71	150.00
9	45.09	2.25	45.09	47.34
Total	---	247.34	1,000.00	1,247.34

"Springfield" Plan

Under this plan, the amount of a loan is divided by the agreed number of installments to find how much the debt is to be reduced by each payment. But the borrower must also pay interest on his outstanding balance each time he pays on his principal. As the amount for interest decreases each time the principal is reduced, the total amount of each installment (on principal and for interest) decreases from one period to the next. Table 4 shows how a plan of this kind works.5/

Under the Springfield plan, the total payment is larger at first than later on, even though reductions of principal remain the same. This partially meets the need for higher payments in the beginning. This plan is therefore particularly suited to farm loans made during periods of prosperity. Under it, a borrower could reduce the principal of his debt faster at first than he would be able to do under the standard amortization plan.

WHEN SHOULD LOANS BE AMORTIZED?

As loans are ordinarily paid from income, the amount and timing of repayments should coincide with those of income. Amortization facilitates repayment of a debt from income; it is feasible when there is some regularity of the income

5/ See footnote 3.

Table 3.- Periodic payment required to pay off a debt of $1, by selected interest rates and by number of payments to be made

Number of periodic payments	Interest rate per period						
	One-half of 1 percent	1 1/2 percent	3 percent	3 1/2 percent	4 percent	4 1/4 percent	4 1/2 percent
	Dollars	Dollars	Dollars	Dollars	Dollars	Dollars	Dollars
1	1.00500000	1.01500000	1.03000000	1.03500000	1.04000000	1.04250000	1.04500000
2	.50375312	.51127792	.52261084	.52640049	.53019608	.53209608	.53399756
3	.33667221	.34338296	.35353036	.35693418	.36034854	.36205965	.36377336
4	.25313279	.25944478	.26902705	.27225114	.27549005	.27711502	.27874365
5	.20300997	.20908932	.21835457	.22148137	.22462711	.22620704	.22779164
6	.16959546	.17552521	.18459750	.18766821	.19076190	.19231731	.19387839
7	.14572854	.15155616	.16050635	.16354449	.16660961	.16815221	.16970147
8	.12782886	.13358402	.14245639	.14547665	.14852783	.15006493	.15160965
9	.11390736	.11960982	.12843386	.13144601	.13449299	.13602944	.13757447
10	.10277057	.10843418	.11723051	.12024137	.12329094	.12483012	.12637882
11	.09365903	.09929384	.10807745	.11109197	.11414904	.11569338	.11724818
12	.08606643	.09167999	.10046209	.10348395	.10655217	.10810349	.10966619
13	.07964224	.08524036	.09402954	.09706157	.10014373	.10170340	.10327535
14	.07413609	.07972332	.08852634	.09157073	.09466897	.09623806	.09782032
15	.06936436	.07494436	.08376658	.08682507	.08994110	.09152043	.09311381
16	.06518937	.07076508	.07961085	.08268483	.08582000	.08741022	.08901537
17	.06150579	.06707966	.07595253	.07904313	.08219852	.08380017	.08541758
18	.05823173	.06380578	.07270870	.07581684	.07899333	.08060681	.08223690
19	.05530253	.06087847	.06981388	.07294033	.07613862	.07776427	.07940734
20	.05266645	.05824574	.06721571	.07036108	.07358175	.07521983	.07687614
21	.05058163	.05586550	.06487178	.06803659	.07128011	.07293083	.07460057
22	.04811380	.05370331	.06274739	.06593207	.06919881	.07086234	.07254565
23	.04613465	.05173075	.06081390	.06401880	.06730906	.06898552	.07068249
24	.04432061	.04992410	.05904742	.06227283	.06558683	.06727631	.06898703
25	.04265186	.04826345	.05742787	.06067404	.06401196	.06571452	.06743903
26	.04111163	.04673196	.05593829	.05920540	.06256738	.06428306	.06602137
27	.03968565	.04531527	.05456421	.05785241	.06123854	.06296736	.06471946
28	.03836167	.04400108	.05329323	.05660265	.06001298	.06175492	.06352081
29	.03712914	.04277878	.05211467	.05544538	.05887993	.06063500	.06241461
30	.03597892	.04163919	.05101926	.05437133	.05783010	.05959825	.06139154
31	.03490304	.04057430	.04999893	.05337240	.05685535	.05863654	.06044345
32	.03389453	.03957710	.04904662	.05244150	.05594859	.05774275	.05956320
33	.03294727	.03864144	.04815612	.05157242	.05510357	.05691064	.05874453
34	.03205586	.03776189	.04732196	.05075966	.05431477	.05613469	.05798191
35	.03121550	.03693363	.04653929	.04999835	.05357732	.05540999	.05727045
36	.03042194	.03615240	.04580379	.04928416	.05288688	.05473220	.05660578
37	.02967139	.03541437	.04511162	.04861325	.05223957	.05409745	.05598402
38	.02896045	.03471613	.04445934	.04798214	.05163192	.05350225	.05540169
39	.02828607	.03405463	.04384385	.04738775	.05106083	.05294350	.05485567
40	.02764552	.03342710	.04326238	.04682728	.05052349	.05241839	.05434315

Table 3.- Periodic payment required to pay off a debt of $1, by selected interest rates and by number of payments to be made - Continued

Number of periodic payments	Interest rate per period					
	4 3/4 percent	5 percent	5 1/2 percent	6 percent	6 1/2 percent	7 percent
	Dollars	Dollars	Dollars	Dollars	Dollars	Dollars
1	1.04750000	1.05000000	1.05500000	1.06000000	1.06500000	1.07000000
2	.53590049	.53780488	.54161800	.54543689	.54926150	.55309179
3	.36548967	.36720856	.37065407	.37410981	.37757570	.38105167
4	.28037592	.28201183	.28529449	.28859149	.29190274	.29522812
5	.22938090	.23097480	.23417644	.23739640	.24063454	.24389069
6	.19544512	.19701747	.20017895	.20336263	.20656831	.20979580
7	.17125735	.17281982	.17596442	.17913502	.18233137	.18555322
8	.15316196	.15472181	.15786401	.16103594	.16423730	.16746776
9	.13912803	.14069008	.14383946	.14702224	.15023803	.15348647
10	.12793699	.12950458	.13266777	.13586796	.13910469	.14237750
11	.11881337	.12038889	.12357065	.12679294	.13005521	.13335690
12	.11124019	.11282541	.11602923	.11927703	.12256817	.12590199
13	.10485950	.10645577	.10968426	.11296011	.11628256	.11965085
14	.09941565	.10102397	.10427912	.10758491	.11094048	.11434494
15	.09472113	.09634229	.09962560	.10296276	.10635278	.10979462
16	.09063531	.09226991	.09558254	.09895214	.10237757	.10585765
17	.08705063	.08869914	.09204197	.09544480	.09890633	.10242519
18	.08388343	.08554622	.08891992	.09235654	.09585461	.09941260
19	.08106766	.08274501	.08615006	.08962086	.09315575	.09675301
20	.07855047	.08024259	.08367933	.08718456	.09075640	.09439293
21	.07628907	.07799611	.08146478	.08500455	.08861333	.09228900
22	.07424846	.07597051	.07947123	.08304557	.08669120	.09040577
23	.07239969	.07413682	.07766965	.08127848	.08496078	.08871393
24	.07071867	.07247090	.07603580	.07967900	.08339770	.08718902
25	.06918513	.07095246	.07454935	.07822672	.08198148	.08581052
26	.06778192	.06956432	.07319307	.07690435	.08069480	.08456103
27	.06649444	.06829186	.07195228	.07569717	.07952288	.08342573
28	.06531016	.06712253	.07081440	.07459255	.07845305	.08239193
29	.06421829	.06604551	.06976857	.07357961	.07747440	.08144865
30	.06320945	.06505144	.06880539	.07264891	.07657744	.08058640
31	.06227550	.06413212	.06791665	.07179222	.07575393	.07979691
32	.06140929	.06328042	.06709519	.07100234	.07499665	.07907292
33	.06060455	.06249004	.06633469	.07027293	.07429924	.07840807
34	.05985574	.06175545	.06562958	.06959843	.07365610	.07779674
35	.05915794	.06107171	.06497493	.06897386	.07306226	.07723396
36	.05850680	.06043446	.06436635	.06839483	.07251332	.07671531
37	.05789843	.05983979	.06379993	.06785743	.07200534	.07623685
38	.05732932	.05928423	.06327217	.06735812	.07153480	.07579505
39	.05679637	.05876462	.06277991	.06689377	.07109854	.07538676
40	.05629675	.05827816	.06232034	.06646154	.07069373	.07500914

Table 4.- Springfield plan: Amortization of $1,000 loan in 10 years if interest is at 5 percent

Year	Unpaid principal at beginning of year	Payment at end of year		
		Interest	Principal	Total
	Dollars	Dollars	Dollars	Dollars
1	1,000	50	100	150
2	900	45	100	145
3	800	40	100	140
4	700	35	100	135
5	600	30	100	130
6	500	25	100	125
7	400	20	100	120
8	300	15	100	115
9	200	10	100	110
10	100	5	100	105
Total	---	275	1,000	1,275

from which repayments can be made. For example, if a farmer's chief enterprise is dairying, monthly payments may be suitable. A farmer whose income is derived chiefly from the sale of one or more crops that mature at about the same time might want to make only one payment a year, perhaps shortly after harvesting is normally completed.

The advisability of amortizing some loans, even by annual payments, might be questioned. For example, a livestock producer who is trying to build up his herd might want to use all available income during a certain period to increase his herd. The need to make periodic payments might seriously inconvenience him before his herd is large enough to produce the income from which to make regular payments. It is not implied that loan repayments should always come out of the income from the enterprise for which the money was borrowed. Sometimes a combination of enterprises on a farm may provide a fairly regular periodic income from which loan repayments can be made.

Income from agriculture is notably irregular. This being true, an amortized loan might easily become delinquent at some time during its term, particularly if the term is a long one. This could occur even if only interest on principal were required to be paid periodically; but the inclusion of an amount for reduction of principal makes delinquency even more likely. If annual income should drop, so that when living expenses have been met not enough is left with which to make a payment, the loan would be delinquent - even though income from another source should make it possible to meet the payment later on.

Because of the year-to-year variability of the income of individual farmers from crops, some lenders permit borrowers to vary the amount of their principal repayments. They may make larger payments than are called for by their amortization schedules in years of high income, and smaller payments in years of low

income. This contribution to flexibility in "fixed" expenses helps to take some of the risk out of farming.

Both the farmer and the lender should recognize this variability of farm income. Farmers should provide for it by maintaining cash reserves from which they can meet such emergencies, and lenders by using - whenever possible - variable-payment provisions that will permit payments to be made in advance in "good" years to offset nonpayments or reduced payments in "poor" years.

Such variable-payment provisions have been used by the Farmers Home Administration, in connection with its farm-ownership loans, since 1946. Borrowers now in the program have repaid about 44 percent more in principal and interest than was required to keep their payments current. Under the present arrangement, a borrower's installment may be deferred in a year of "below-normal" income, provided he has made advance payments. A "normal" year is described in the promissory note signed by the borrower as one in which income "equals an amount sufficient to meet the amount payable for that year under the above schedule, and also to meet reasonable farm operating expenses, normal capital replacements, and usual family living expenses ..." For any year for which the borrower's income is determined to be below normal, "the amounts previously paid as advance payments may be used to supplement the amount available from that year's operations for application on the annual installment due ..." Interest continues to accumulate even if a payment is deferred.

A borrower's payment to the Farmers Home Administration may be deferred, therefore only after it has been determined that he has had a year of below-normal income, and then only if he is ahead on his amortization schedule. A borrower may not miss a payment in a year of normal or above-normal income without being in arrears. Under a previous arrangement, a borrower was not required to be ahead with his payments in order to be permitted to postpone an installment.

Under the variable-payment plan of the Farmers Home Administration, advance payments are not set aside, as they are by the Farm Credit Administration, in a separate "future payments fund." The advance payments to FHA are credited immediately toward reduction of principal. Whether a borrower is ahead or lagging in his payments is determined by comparing his unpaid principal with the balance that would still be due, according to a standard amortization schedule covering his loan.

Under the arrangement in effect in the Federal Land Bank System, in good years borrowers may either (1) make installment payments in advance, or (2) have the advance payment credited to a future payment fund. If a borrower elects to have his advance payment credited to the future payment fund, its acceptance is subject to such terms and conditions as may be agreed upon by the Federal land bank and the borrower. An agreement of this kind specifies the rate of interest to be allowed on such funds, which in most land banks is the same as the rate that the borrower pays on his loan. At his option the borrower may draw on this fund to make installment payments in the future.

In connection with the servicing of delinquent loans and sales contracts of foreclosed farms, the Federal land banks may include variable-payment provisions in their mortgage contracts.

Other lenders also have established practices of extending and deferring delinquent payments. They, too, have found that delinquency for a year or two does not necessarily mean that a loan is unsound. Most of these arrangements are discretionary on the part of lenders and ordinarily do not appear in their loan contracts. Therefore, they give a borrower no legal protection. Whether or not an advance payment by a borrower is associated with specified installments, by number, the advance payment does not prevent him from becoming delinquent if a later installment is missed.

Lenders might consider inserting variable-payment provisions in their loan contracts, or at least provide for the reamortization of a loan over its remaining term, after a lump-sum payment has been made, so that a lesser periodic payment would be required from then on. The highly variable incomes of farmers might justify permitting them to make lump-sum debt reductions without restrictions or penalty, even though such provisions might necessitate some later increase in interest rates. But it should be recognized that any farmer with a mortgage on his place may set aside his own "future payments fund" from current income, to provide for years of low income, when payments would be difficult. A fund of this kind might also serve as an all-purpose reserve, for use in case of sickness or other unforeseen emergency.

From a lender's viewpoint, amortization of a debt by regular payments is usually desirable. A gradual repayment of the principal of a loan makes the loan more secure. At the time a loan is made it may seem fairly clear when and how farm income will be available for debt repayment in the immediate future. The longer the term of a loan, however, the more uncertain its repayment becomes. If the loan is reduced periodically, the risk of future difficulty in repaying it is decreased.

Farmers, like other borrowers, sometimes overlook the necessity of putting away small amounts of money periodically in order to have cash enough to meet the full amount of a loan when it becomes due. Amortization is a substitute for establishing what might be called a "sinking fund" to repay a debt. This attention to repayment also decreases the risk of the loan to the lender, as pointed out in the preceding paragraph. It may also permit a lender to lend more as an amortized loan, on the same security, than would be possible otherwise. For example, an amortized loan on real estate at 80 percent of its value may be as safe over a 20-year term as an unamortized loan for 50 to 60 percent of value over the same period. The average principal outstanding on an amortized loan for 80 percent of value over 20 years, at 4 percent interest, is only 47 percent of the initial valuation.

Many lenders will find amortization of a loan impracticable because they have neither the desire nor the facilities to reinvest or otherwise to handle the small driblets of principal repaid on an amortized basis. They may prefer to invest their money for a substantial period with intervening transactions except payment of interest. They may even wish to let interest accumulate and compound. But this method of financing is not popular in agriculture, even though it may have many logical applications. Some lenders are also bothered by the computations that are required to set up and service amortized loans.

For many uses of credit in farming, the extra income derived comes in periodically during the life of the loan. This is true of most investments in such items as livestock and machinery. By amortizing such loans, a farmer automatically pays off his debt, so that at the end of the term he can replace equipment or other capital goods by obtaining another loan. If he does not do this, he may find that he still owes for capital goods after they are worn out. If so, he has let his equity disappear by "living off his principal." This occurs sometimes when a farmer does not account for all of his costs. Even though a loan is not obtained, if the income resulting from the use of capital is insufficient to repay the investment, the investment was not good. Amortization of a loan includes periodic reduction of the principal and, if all payments are made promptly, it assures full repayment of the debt by the end of the term.

Goods bought on credit for use in the home, such as electric refrigerators, home freezers, or hot-water heaters, usually do not provide farmers with additional income from which the debt may be paid. Nevertheless, the investment may be repaid in health and happiness. Benefits from such purchases generally accrue continuously and, if farmers have a regular source of income, either from farming or from other sources, amortization would appear to be a logical method of debt repayment.

The need for credit to finance the purchase of land differs from the need for credit to buy capital equipment. Repayments of principal on a loan secured by real estate often represent an increase in a farmer's equity or savings instead of an offset to the annual cost of depreciation and obsolescence. The desirability of amortized loans over long periods for purchase of land is therefore related to the need or desire of a farmer to increase his equity as he goes along. For a young farmer who is building up an economic unit, periodic payments might slow his progress toward that goal. For example, he might need livestock or machinery more than he needs to build up his equity in land and buildings. But once a farmer has attained an economic unit, regular repayments of principal could develop good savings habits and enable him to pay off a debt more easily than if he had to establish his own sinking fund to pay it off at a specified future date.

Some farmers who have paid for their farms in full or who have high equities in them may need cash but they may not be interested in further savings. The money might be needed for improvements to make living more enjoyable, for educating their children, for payment of doctors' bills, or for travel. An unamortized loan, secured by a mortgage, will serve these purposes. In a high-income year, these farmers may decide to pay off all or part of their debts, or they may merely pay interest on them. But if they obtain amortized loans, the amortization process requires them to build back their equities gradually.

Loans based primarily on income prospects, rather than on valuation of property pledged as security, may be especially needed by some young farmers who have uneconomic units. Prospective future incomes may justify some loans to develop farms, even though the security itself is inadequate by the usual standards at the time the loans are closed. Many otherwise uneconomic units, if properly developed, could earn more than the interest on the additional investment. Such units may not be "adequate security" for the needed credit, yet future incomes from the properly improved and enlarged units would more than carry the interest and retire the loans.

In some periods current land values and the value of capital goods are not warranted by probable future incomes. During such periods improvement and enlargement of uneconomic units may not be feasible. But there are periods in which the reverse is true. The security of any credit extension depends upon the point in the business cycle at which the loan is made. A loan made on the upturn, for a given amount on a particular property, has a greater chance of success than if it were made on the downturn.

STANDARD AMORTIZATION CALCULATIONS

Other Uses

Amortization is useful when applied to a gradual liquidation of a loan and also when applied to certain other financial problems of the farmer. The amount of a loan that can be amortized by a given periodic payment is merely the present value of these future payments. If the prospective purchaser of a machine knows, for instance, how much additional annual return he can expect from the machine and how long it will last, he can determine the maximum amount he can afford to pay for it. Suppose a farmer has decided that a certain machine will add $100 a year to his net income for the next 10 years. The present value of this series of future net incomes, at 5 percent interest, is $772.17.6/ He could afford to pay $772.17 for the machine and to pay interest on his investment at 5 percent and also have his investment returned to him in 10 years at $100 a year. If he had to borrow the money, he could pay 5 percent interest on the loan and repay the loan in 10 years from the income.

A farmer who is about to retire may wish to invest his money, possibly from the sale of his farm, so as to be assured of a periodic income for a selected number of years. He may have reached an age at which he wants to live on his principal as well as his interest as his periodic income is received. As an example, suppose a farmer sells his farm for $25,000, and receives $5,000 as a down payment. He wants to receive the remaining $20,000 in monthly payments during the next 20 years. If interest is at 5 percent a year, he would receive $130.77 a month.7/ If he spent the money as he received it, this farmer would gradually exhaust his principal. Reduction of principal would be small at first, but it would grow larger. The part consisting of interest would be large at first; it would become smaller later on. At the end of 20 years, the principal would be gone.

Some farmers who have no children or who have other assets to leave as an inheritance for their children, may find this type of investment suitable for their needs. If part of the monthly income were saved as it came in, accumulations could be reinvested, and the estate would include additional assets for distribution to heirs.

Methodology

Under the standard plan, the amount of the periodic payment is always the same. This periodic payment is so calculated that it will exactly pay off the debt, with interest, during the term of the loan. Payments need not be made

6/ See example 4 under case 1, p. 15.
7/ See example 4 under case 2, in appendix (p. 18).

annually in connection with loans secured by real estate, although many of them are so made.

One of two methods is used to determine the amount of the periodic payment and (if an amortization table is to be constructed) the interest rate to be applied each time a payment is made. The difference in these methods arises from the fact that when an interest rate is quoted, say 5 percent, it is sometimes meant to be an "effective" rate per year. At other times it is meant to be a "nominal" rate. In the latter case, the rate is quoted only to afford a convenient basis for arriving at the semiannual, quarterly, or monthly interest rate that is actually applied when payments are made. For example, a rate of 4 1/2 percent may be quoted when payments are to be made monthly. If the lender uses one-twelfth of the quoted rate, or 0.00375 (which is three-eighths of 1 percent), in computing the interest due each month, the quoted rate of 4 1/2 percent is a nominal rather than an effective rate. A borrower should understand that in a case like this he pays interest at a rate higher than the one quoted. In the example, he would be paying 4.594 percent a year, rather than 4 1/2 percent. When interest is collected more than once a year, it is compounded. As this "nominal" method of charging interest is more commonly used, it is described first, as "case 1." The other method of charging interest is described in the appendix as "case 2."

It is frequently impossible to tell from the way a contract is worded which of these two methods should be applied. If the rate is quoted at, say, 5 percent compounded semiannually, quarterly, or monthly, case 1 applies; but this method is often applied in practice even if the rate is quoted as "5 percent per annum." In such cases, the only way to tell which method to apply is to know the rate the lender intends to charge on the unpaid principal each time a payment is made.

Case 1 Problems

Most amortization problems fall in this category, as annual interest rates are usually interpreted in such a way that case 1 applies. For example, if payments are made annually and the rate of interest is 5 percent a year, the problem is handled under case 1. Other problems coming under case 1 arise:[8/]

1. If payments are made semiannually and if the interest rate is half of the rate quoted per semiannual period. For example, if the interest rate is quoted as "5 percent compounded semiannually," a rate of 2 1/2 percent applies semiannually. There are 6 months in each payment period and also in each interest period.

2. If payments are made quarterly and if the interest rate is one-fourth of the rate quoted per quarterly period. For example, if the interest rate is quoted as "5 percent compounded quarterly," a rate of 1 1/4 percent applies quarterly. There are 3 months in each payment period and also in each interest period.

8/ In each example the payment period covers the same number of months as the interest period.

3. If payments are made monthly and if the interest rate per payment period is one-twelfth of the rate quoted. For example, if the interest rate is quoted as "6 percent compounded monthly" a rate of one-half of 1 percent applies monthly. Each payment period and each interest period consists of 1 month.

Instructions

1. Determine the number of payments to be made and the applicable interest rate per payment period.

2. Find the appropriate factor for (periodic payment on) a loan of $1 in table 3 and multiply it by the number of dollars borrowed to find the amount of the periodic payment.

Examples

1. If payments are to be made annually for 10 years, and the interest rate is 5 percent a year, there are 10 payment periods and the interest rate is 5 percent per payment period.

The appropriate factor for $1 is 0.12950458, found in the 5-percent column of table 3 opposite 10 periods; so for a loan of $1,000 the payment would be $129.50 a year. (See table 1.)

2. If payments are to be made semiannually for 10 years and the interest rate is 5 percent compounded semiannually, there are 20 payment periods in the term of the loan and the interest rate per payment period is 2 1/2 percent.

The appropriate factor for $1 is 0.06414713, found in the 2 1/2 percent column of table 3 opposite 20 periods; so for a loan of $1,000 the payment would be $64.15 every 6 months for 10 years. The problem is handled as though payments were to be made annually for 20 years and the interest rate were 2 1/2 percent annually. A few entries from the amortization table are shown below:

Period	Unpaid principal at beginning of period	Payment at end of period		
		Interest 1/	Principal 2/	Total
(1)	(2)	(3)	(4)	(5)
	Dollars	Dollars	Dollars	Dollars
1	1,000.00	25.00	39.15	64.15
2	960.85	24.02	40.13	64.15
3	920.72			
20	62.59	1.56	62.59	64.15

1/ 0.025 x column 2.
2/ Column 5 minus column 3.

3. If payments are to be made quarterly for 10 years and the interest rate is 6 percent compounded quarterly, there are 40 payment periods in the term of the loan and the interest rate per payment period is 1 1/2 percent.

The appropriate factor for $1 is 0.03342710, found in the 1 1/2 percent column of table 3 opposite 40 periods; so for a loan of $1,000 the payment would be $33.43 every 3 months. The problem is handled as though payments were to be made annually for 40 years and the interest rate were 1 1/2 percent annually. A few entries in the amortization table are shown below:

Period	Unpaid principal at beginning of period	Payment at end of period		
		Interest 1/	Principal 2/	Total
(1)	(2)	(3)	(4)	(5)
	Dollars	Dollars	Dollars	Dollars
1-------	1,000.00	15.00	18.43	33.43
2-------	981.57	14.72	18.71	33.43
3-------	948.14			
40------	32.94	.49	32.94	33.43

1/ 0.015 x column 2.
2/ Column 5 minus column 3.

4. If a machine would add $100 a year to net income for the next 10 years, how much could you afford to borrow at 5 percent a year to pay for it?

The periodic payment is $100, the period is 10 years, and the interest rate is 5 percent a year. What is the present value of this series of future incomes?

The appropriate factor for a loan of $1 is found in the 5-percent column of table 3 opposite 10 periods. It is 0.12950458. So if a loan of $1 could be paid off with annual payments of $0.12950458, what size loan could be paid off with $100 installments? Dividing $100 by 0.12950458, we find that we could borrow $772.17 and expect to repay it at $100 a year for 10 years if the interest rate is 5 percent a year. Stated another way, $772.17 is the present value of a series of $100 payments, made annually for 10 years, if the interest rate is 5 percent a year.

APPENDIX

Case 2 Problems

Occasionally, the annual rate of interest quoted by a lender is intended to be an effective rate per year, even on contracts payable semiannually, quarterly, or monthly. In such cases, the problem of determining the amount of each periodic payment, the interest rate per payment period, and the construction of an amortization table must be solved by the methods described below.

If interest is charged more often than once a year, interest is compounded and the effective rate is higher than the quoted rate. For example, if interest is collected monthly, from monthly payments, the effective annual rate is more than 12 times the monthly rate used. A lender who quotes a (nominal) rate of 5 percent a year, and who charges interest at five-twelfths of 1 percent a month (one-twelfth of 5 percent), is actually charging an effective rate of 5.12 percent a year, rather than 5 percent. Therefore, if he intends to charge only 5 percent a year, he must charge the borrower something less than five-twelfths of 1 percent a month (actually his monthly rate would be 0.00407412).9/

Other problems that may be handled under case 2 are not solved here. They occur rather infrequently. One such problem would be where the payments are made monthly but the interest rate is quoted as "3 percent compounded semiannually." Each payment period covers 1 month and each interest period 6 months (so that 1 1/2 percent interest applies semiannually). In all case 2 problems the number of months in the payment period differs from that in the interest period.

Instructions

(Use tables 3, 5, and 6 if you wish to construct the amortization table)

1. Find, from table 3, the amount that would need to be paid per $1 borrowed if payments were made annually.

2. Multiply this figure by the amount of the loan.

3. This product slightly overstates the sum of the payments that will actually have to be made in one year; so multiply it by the appropriate correction factor from table 5 to arrive at the exact amount of the periodic payment.

4. To construct an amortization table, find the appropriate interest-rate equivalent per payment period from table 6.

Examples

1. Payments are to be made semiannually for 10 years, and the interest rate is 5 percent a year. The amount of the loan is $1,000.

If payments were made only once a year, they would be $0.12950458 per $1 loaned. This figure is found in the 5-percent column of table 3 opposite 10 periods. On $1,000, the payment would be $129.50 a year.

The appropriate correction factor for semiannual payments is 0.49390153. It is found in the 5-percent column of table 5 opposite 2 payment periods a year. Multiplying $129.50 by 0.49390153 gives $63.96 as the semiannual payment.

To construct an amortization table, find the interest-rate equivalent per payment period from table 6. It is $0.02469508, and is found in the 5-percent column of table 6 opposite two payment periods a year. A few entries in the amortization table are shown below:

9/ This figure may be found in the 5-percent column of table 6 opposite 12 payments a year.

Period	Unpaid principal at beginning of period	Payment at end of period		
		Interest 1/	Principal 2/	Total
(1)	(2)	(3)	(4)	(5)
	Dollars	Dollars	Dollars	Dollars
1---------	1,000.00	24.70	39.26	63.96
2---------	960.74	23.73	40.23	63.96
3---------	920.51			
20--------	62.42	1.54	62.42	63.96

1/ 0.02469508 x column 2.
2/ Column 5 minus column 3.

2. A $1,000 loan is to be paid off in 10 years by quarterly payments. If interest is at 5 percent a year, what is the amount of each quarterly payment?

If payments were to be made only once a year, they would be $0.12950458 per $1 loaned. This figure is found in the 5-percent column of table 3 opposite 10 periods. On $1,000, $129.50 would be paid each year.

The appropriate correction factor for quarterly payments is 0.24544469. It is found in the 5-percent column of table 5 opposite 4 payment periods a year. Multiplying $129.50 by 0.24544469 gives $31.79 as the quarterly payment.

To construct an amortization table, find the interest rate equivalent per payment period from table 6. It is $0.01227224, and is found in the 5-percent column opposite 4 payment periods a year. A few entries in the amortization table are shown below:

Period	Unpaid principal at beginning of period	Payment at end of period		
		Interest 1/	Principal 2/	Total
(1)	(2)	(3)	(4)	(5)
	Dollars	Dollars	Dollars	Dollars
1---------	1,000.00	12.27	19.52	31.79
2---------	980.48	12.03	19.76	31.79
3---------	960.72			
40--------	31.40	.39	31.40	31.79

1/ 0.01227224 x column 2.
2/ Column 5 minus column 3.

3. A $1,000 debt is to be amortized in 10 years by monthly payments. If the rate of interest is 4 1/2 percent a year, how much are the monthly payments?

If payments were made only once a year, they would be $0.12637882 per $1 of debt. This figure is found in the 4 1/2 percent column of table 3 opposite 10 periods. On $1,000, the payment would be $126.38 a year.

The appropriate correction factor for monthly payments is 0.08166243. It is found in the 4½-percent column of table 5 opposite 12 payment periods per year. Multiplying $126.38 by 0.08166243 gives $10.32 as the monthly payment.

To construct an amortization table, find the interest-rate equivalent per payment period from table 6. It is $0.00367481, and is found in the 4 1/2 percent column of table 6 opposite 12 payment periods a year. A few entries in the amortization table are shown below:

Period	Unpaid principal at beginning of period	Payment at end of period		
		Interest 1/	Principal 2/	Total
(1)	(2)	(3)	(4)	(5)
	Dollars	Dollars	Dollars	Dollars
1---------	1,000.00	3.67	6.65	10.32
2---------	993.35	3.65	6.67	10.32
3---------	986.68			
120--------	10.28	.04	10.28	10.32

1/ 0.00367481 x column 2.
2/ Column 5 minus column 3.

4. A farmer wishes to sell his farm and move into town. A buyer will pay him $25,000 for it, making a $5,000 down payment. The balance is to be paid monthly over a period of 20 years, with interest at 5 percent a year. What will the monthly payment be?

If payments were made only once a year, this farmer would receive $0.0802459 annually for each $1 of balance owed. This figure is found in the 5-percent column of table 3 opposite 20 periods. On $20,000, he would receive $1,604.85 a year.

The appropriate correction factor for monthly payments is 0.08148248. It is found in the 5-percent column of table 5, opposite 12 payments a year. Multiplying $1,604.85 by 0.08148248 gives $130.77 as the monthly payment.

If this farmer spent the $130.77 as it came in, he would be exhausting his principal along with the interest earned by it. In the early years, most of the monthly payment would be interest, with little exhaustion of principal. In the later years, most of the monthly income would be made up of principal. At the end of 20 years, his principal would be gone. This situation may fit the needs of some farmers who have no children or who have other assets as an inheritance for their children.

5. A farmer decides that, after his death, the proceeds of a $10,000 life insurance policy be made payable to his widow as a monthly income rather than as a lump sum. He figures she will need $75 a month, in addition to other income, for living expenses. If an interest rate of 3 percent a year applies to the policy, how long will the monthly payments continue? In other words, at $75 a month, how long will the $10,000 last under a monthly income settlement option?

The appropriate correction factor for monthly payments, when the rate of interest is 3 percent a year, is $0.08220899 per $1 of principal. This figure is found in the 3-percent column of table 5, opposite 12 payments a year. Multiplying it by $10,000 gives $822.09.

Table 5.- Correction factors to be used in determining amount of periodic payment when annual interest rate is quoted but more than one payment a year is to be made

Payments per year	Annual interest rate				
	2 1/2 percent	3 percent	3 1/2 percent	4 percent	4 1/2 percent
	Correction factor				
2	0.49691346	0.49630522	0.49569993	0.49509757	0.49449811
3	.33059350	.33005447	.32951834	.32898510	.32845470
4	.24768985	.24723573	.24678417	.24633516	.24588868
6	.16495662	.16462073	.16428684	.16395492	.16362496
12	.08239345	.08220899	.08202568	.08184349	.08166243
	5 percent	5 1/2 percent	6 percent	6 1/2 percent	7 percent
2	.49390153	.49330780	.49271690	.49212880	.49154348
3	.32792714	.32740237	.32688037	.32636113	.32584460
4	.24544469	.24500317	.24456410	.24412746	.24369321
6	.16329692	.16297080	.16264657	.16232422	.16200372
12	.08148248	.08130362	.08112584	.08094914	.08077351

The $75 a month which this farmer wants his widow to receive is 9.123 percent of the $822.09, or 0.09123, expressed as a decimal fraction. We have computed, at 3 percent a year, an annual payment per $1 of policy proceeds - taking into consideration the fact that payments are to be made monthly. How many years will the payments continue?

Looking for 0.09123 in the 3-percent column of table 3, we find it to be about halfway between 13 and 14 years.10/ Therefore, the $10,000 would provide payments of $75 a month for 162 months, beginning one month after the death of the farmer.

Dividing $10,000 by $75, we find that, without interest, the money would last only 133 1/3 months. But because of interest, the widow would receive payments for about 29 additional months.

10/ By interpolation, the 0.09123 is opposite 13.49 years.

Table 6.- Interest rates per payment period that are equivalent (if compounded for a year) to specified effective rates, by number of payments to be made each year

Payments per year	Annual interest rate				
	2 1/2 percent	3 percent	3 1/2 percent	4 percent	4 1/2 percent
	Interest rate per payment period				
2	0.01242284	0.01488916	0.01734950	0.01980390	0.02225242
3	.00826484	.00990163	.01153314	.01315941	.01478046
4	.00619225	.00741707	.00863745	.00985341	.01106499
6	.00412392	.00493862	.00575004	.00655820	.00736312
12	.00205984	.00246627	.00287090	.00327374	.00367481
	5 percent	5 1/2 percent	6 percent	6 1/2 percent	7 percent
2	.02469508	.02713193	.02956302	.03198837	.03440804
3	.01639636	.01800713	.01961282	.02121347	.02280912
4	.01227224	.01347518	.01467385	.01586828	.01705853
6	.00816485	.00896340	.00975880	.01055107	.01134026
12	.00407412	.00447170	.00486755	.00526169	.00565415